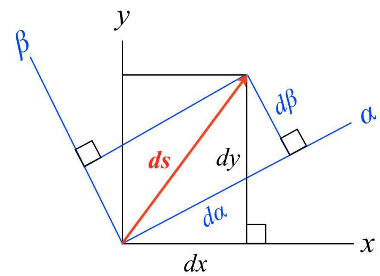
IAU Symposium 338 Gravitational Wave Astrophysics Baton Rouge, Louisiana 2017

How Einstein's Theory of Relativity gives us $E = m\,c^2$ and the Atomic Bomb

The first step:

We remind ourselves about Pythagorean invariance under a two - dimensional rotation in space:



$$\boldsymbol{d\alpha^2 + d\beta^2 = ds^2 = dx^2 + dy^2}$$

And then—the crucial next step:

Suppose that your sister is moving along your x-axis at a constant speed v

and that during that motion, your sister snaps her fingers twice, $d\tau$ seconds apart —

for you, those two finger-snaps are separated in space by dx and are separated in time by dt

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RELATIVITY: → **Pythagoras, Einstein, and Minkowski assert that $d\tau$ and dt are related by:**

your Sister → $d\alpha^2 + d\beta^2 + d\gamma^2 - d\tau^2 = ds^2 = dx^2 + dy^2 + dz^2 - dt^2$ ← **You**

$$0^2 + 0^2 + 0^2 - d\tau^2 \;=\; dx^2 - 0^2 - 0^2 - dt^2$$

$d\tau^2 = dt^2 - dx^2$ ← **if they are right (and experiment proves that they are) time slows down for your sister!**

but $dx = v\,dt$ **so** $d\tau^2 = (1 - v^2)\,dt^2$ **and so** $(\frac{dx}{dt} =)\;\; v \le 1\,!$ ← **and so the universal speed-limit is ONE**

(for a photon $v = 1$, so $d\tau = 0$ — and so, in its own rest frame no photon ever exists—just ask any photon!)

$d\tau^2 = dt^2 - dx^2 \;\;\rightarrow\;\; 1 = (\frac{dt}{d\tau})^2 - (\frac{dx}{d\tau})^2 \equiv$ (**re *time:* Newton will help us** *!* $)^2$ − (**re *space:* we're OK, ∼ "as is"**$)^2$

$(\gamma \equiv)\;\frac{dt}{d\tau} = \sqrt{\frac{1}{1-v^2}}\;\;\;(= 1 + \frac{1}{2}v^2 + \frac{3}{8}v^4 + \cdots$ **←expansion, from Newton)** • • $\frac{dx}{d\tau} = \frac{dx}{dt}\cdot\frac{dt}{d\tau} = v\,\frac{dt}{d\tau} = v\,\gamma$

and so: $1 = (1 + \frac{1}{2}v^2 + \frac{3}{8}v^4 + \cdots)^2 - (\gamma\; v)^2$

$1 = [1 + \frac{1}{2}(\frac{v}{c})^2 + \frac{3}{8}(\frac{v}{c})^4 + \cdots]^2 - [\gamma(\frac{v}{c})]^2$ **where** $c = 1$ **or (by convention) = 299,792,458** m/s

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$m^2c^2 = \frac{1}{c^2}(m\,c^2 + \frac{1}{2}m\,v^2 + \cdots)^2 - (\gamma\,m\,v)^2$ ← **please notice Newton's** $E = \frac{1}{2}m\,v^2$ **and** $p = m\,v\,!$

$\boldsymbol{m^2c^2 = \frac{E^2}{c^2} - p^2}$ **and so, beyond Newton:** → $E = m\,c^2 + \frac{1}{2}m\,v^2 + \cdots$ **and** $p = \gamma\;m\,v$

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and now: when we consider the case $v = 0$, we find the most famous equation in human history:

$$E = m\,c^2$$

which gives the Atomic Bomb: $n + {}^{235}U \rightarrow {}^{236}U \rightarrow {}^{92}Kr \,+\, {}^{141}Ba + \langle E = [m_U - (m_{Kr} + m_{Ba})] \times c^2\rangle + (3\,n\;\text{ out} \rightarrow)$

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