In 1905 Albert Einstein discovered — via two clumsy postulates—the Theory of Relativity, changing physics forever.

Then, in 1908, Einstein’s former teacher, Hermann Minkowski, re-derived Einstein’s relativity—but from a single brilliant Pythagorean postulate—and made the most important discovery—ever—in physics: of the nature of time.

**Pythagoras:** We claim that \( ds^2 = dx^2 + dy^2 \) ... but we prove it by eye, just comparing these two drawings:

In 3-D, Pythagoras becomes \( ds^2 = dx^2 + dy^2 + dz^2 \)  

**Minkowski:** Einstein’s idea really was that \( ds^2 = dx^2 + dy^2 + dz^2 - dt^2 \) is the (hypotenuse)**\(^2** of two events, with \( ds \) being the separation in space-and-time (both for you and for any other observer) of those two events.

Yes! similar to Pythagoras—but with a minus sign distinguishing time from being just one more space dimension.

**That is the Theory of Relativity:** the rest, is the use of simple math to work out the incredible consequences:

**First step!** Suppose you are stationary—and you erect a coordinate system \((x, y, z, t)\) —with yourself at the origin.

Now, suppose that you see your twin sister redhead moving at constant velocity \( v \) along your \( x \) axis.

Redhead erects her own (moving, at \( v \)) coordinate system \((x', y', z', t')\), choosing her \( x' \) axis to lie atop your \( x \) axis.

Redhead snaps her fingers twice as she moves—and each of you record the times of those two events.

Redhead easily notes those times but you need two assistants, on the \( x \) axis at the two points where the snaps occur.

Your two (stationary) assistants report that redhead’s two finger snaps occurred at \((x_1, 0, 0, t_1)\) and \((x_2, 0, 0, t_2)\)  

while redhead herself recorded that they occurred at \((0, 0, 0, t'_1)\) and \((0, 0, 0, t'_2)\)

The changes \( dx \) and \( dt \) (between the two finger snaps) that you record are \( dx = x_2 - x_1 \) and \( dt = t_2 - t_1 \)

The changes \( dx' \) and \( dt' \) redhead (moving at \( v \)) finds are \( dx' = x'_2 - x'_1 = 0 - 0 = 0 \) and \( dt' = t'_2 - t'_1 = d\tau \)

**\( d\tau \) will be key!** Also—redhead is, of course, always at \( x' = 0 \) in her own moving frame: that is why I set \( dx' = 0 \).

For both of you, the two finger snaps occur on the \( x \) axis, so: \( dy = 0 \) and \( dy' = 0 \); also, \( dz = 0 \) and \( dz' = 0 \)

**This was Einstein’s relativity claim:** \( + dx^2 + dy^2 + dz^2 - dt^2 = ds^2 = + dx'^2 + dy'^2 + dz'^2 - dt'^2 \) so the \( ds^2 \) for the two “finger-snap” events—for you and for redhead—are equal: \( dx^2 + dy^2 + dz^2 - dt^2 = 0 + 0 + 0 + dt'^2 \)

**Einstein had found** the value of \( d\tau \) compared with \( dt \) ! Newton—incorrectly—had claimed time to be universal!

Tidying (and rearranging) our equation, we have, finally, the heart of relativity — that for \( v \), \( dt'^2 = dt^2 - dx^2 \)
the mass \( m \) of a particle is merely a dimensionless number—an invariant—and invariants, well— never vary

\[
dt^2 = dt^2 - dx^2, \text{ so } \Delta t^2 \neq dt^2 \quad \text{Between the two snaps, you see redhead travel } \frac{dx}{dt} = \text{velocity} \times \text{time} = v \, dt, \text{ so } \Delta t^2 = dt^2 - v^2 dt^2 = dt^2 (1 - v^2). \quad \text{Since } \Delta t^2 \text{ is necessarily positive — no velocity } v \text{ can ever exceed 1.}
\]

Distances are in light-years, and time-intervals are in years: light goes at 1; your 60 mph car goes at 0.000,000,089

Since \( \Delta t^2 = dt^2 - dx^2 \), 1 = \( \frac{dt^2}{dx^2} \frac{dx^2}{dt^2} \). We now define 1) \( \gamma \equiv \frac{dt}{dx} \) and 2) \( u \equiv \frac{dx}{dt} \) \( \rightarrow u \text{ is redhead's speed, but for you, redheads' speed is } v \equiv \frac{dx}{dt} \). You both agree on the distance she goes, but disagree on how long it takes.

You are both right: your twin ages more slowly than you do:

\[
dt^2 \text{ is } < dt^2 \text{ by the (usually tiny) amount } dx^2. \quad \text{Next, notice that } \gamma = \frac{dt}{dx} = \frac{\frac{dt}{dx}}{\frac{dx}{dt}} = \frac{u}{v} \quad \text{so } \quad u = \gamma v \text{ and } dt = \frac{dt}{\gamma}
\]

From our two definitions \( \left(\frac{dt}{dx}\right)^2 - \left(\frac{dx}{dt}\right)^2 = 1 = \gamma^2 - u^2 \quad \text{so } 1 = \gamma^2 - u^2 \text{ and } \gamma^2 = \frac{1}{1-u^2} = \left(\frac{dt}{dx}\right)^2 \)

so \( \gamma = \frac{dt}{dx} = \sqrt{1-u^2} = 1 + \frac{1}{2} \, v^2 + \frac{3}{8} \, v^4 + \cdots \) \( \left(\text{last step: a binomial expansion (Isaac Newton): and so } \gamma \approx 1 + \frac{1}{2} \, v^2 \right) \)

(binomial expansion yields, for \( [\text{huge}] \) \( v = 0.1, \text{using just those first 2 terms: } 1.005 \approx 1.005 \times 0.089 = \gamma \))

\[ dt = dt \sqrt{1 - v^2} \text{ has drastic consequences for photons, for which } v = 1 \text{ and so } dt = 0 \text{ and so photons never exist.} \]

Massive particles (quarks, electrons, ...) get their masses by coupling to the Higgs field, but between such couplings for them, \( dt = 0 \), and so no particles (in their own reference frames) ever have any existence in time.*

\[ 1 = \gamma^2 - u^2 = \left(1 + \frac{1}{2} \, v^2 + \frac{3}{8} \, v^4 + \cdots \right)^2 - u^2 \text{ and now multiply by } m^2 \left(\text{first appearance of redhead's mass } m!\right) \]

\[ m^2 = \left[ m + \frac{1}{2} \, mv^2 + \frac{3}{8} \, mv^4 + \cdots \right]^2 - \left(\gamma v m\right)^2 \quad \text{typical } v \text{'s are } \ll 1, \text{so the } 1^\text{st} \text{ term in the expansion is enormous!} \]

Our last equation is a triumph for Albert Einstein, for it connects Newton’s expressions for energy and momentum:

Newton said that energy was \( \frac{1}{2} \, mv^2 \) — but Einstein reveals energy, actually, to be: \( E = m + \frac{1}{2} \, mv^2 + \frac{3}{8} \, mv^4 + \cdots \)

Newton said that momentum was \( mv \) — but Einstein reveals momentum, actually, to be: \( p = \gamma mv \) (\( \gamma \) ranges 1 to \( \infty \))

If a particle of mass \( m \) is not moving at all, \( v = 0 \) and its momentum (for both Newton and Einstein) is zero. But, while its energy is zero for Newton, for Einstein the particle still has a hidden, enormous, residual energy \( m \). That is the origin of the most famous equation (except for that of Pythagoras) in human history: \( E = mc^2 \)

Despite time being just one more dimension, we humans have developed the habit of using different units for time (seconds) than what we use for spatial distances (meters). There is a conversion factor between these units (just like there is a conversion factor of 3 feet/yard). By (historical) rather ridiculous choice, our conversion factor is (brace yourself!): \( 299792458 \text{ meters/second } \equiv c \text{ m s}^{-1} \)

\[ m^2 = \left[ m + \frac{1}{2} \, m \left(\frac{v}{c}\right)^2 + \frac{3}{8} \, m \left(\frac{v}{c}\right)^4 + \cdots \right]^2 - \left[ \gamma m \left(\frac{v}{c}\right)^2 \right]^2 \quad \text{but that's only because we forced in that (totally arbitrary) big number } c \]

\[ m^2 c^2 = \frac{c^4}{c^2} \left[ m + \frac{1}{2} m \left(\frac{v}{c}\right)^2 + \frac{3}{8} m \left(\frac{v}{c}\right)^4 + \cdots \right]^2 - \left[ \gamma m \left(\frac{v}{c}\right)^2 \right]^2 \quad \text{I've only multiplied each of the three terms by } c^2 \]

\[ m^2 c^2 = \frac{1}{c^2} \left[ m c^2 + \frac{1}{2} \, m \, v^2 + \frac{3}{8} \, m \left(\frac{v}{c}\right)^4 + \cdots \right]^2 - \left(\gamma v m\right)^2 \quad \text{Einstein links Newton’s } E \text{ and } p \]

We re-define energy to be \( E \equiv mc^2 + \frac{1}{2} \, mv^2 + \frac{3}{8} \, m \left(\frac{v}{c}\right)^4 + \cdots \) and we re-define linear momentum to be \( p \equiv \gamma mv \)

So! \( m^2 c^2 = E^2/c^2 - p^2 \) or \( E^2 = (mc^2)^2 + (cp)^2 \) and! ... when \( p = 0 = \gamma mv \), then, at last! \( E = mc^2 \)

The A-Bomb: \( ^{235}\text{U is hit by a neutron, } n: \quad n + ^{235}\text{U} \rightarrow ^{236}\text{U} \rightarrow ^{92}\text{Kr} + ^{141}\text{Ba} + \left[ E = (m_n - m_{^{235}\text{U}} - m_{^{92}\text{Kr}}) \times c^2 \right] + (3 \text{ n out!}) \)

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