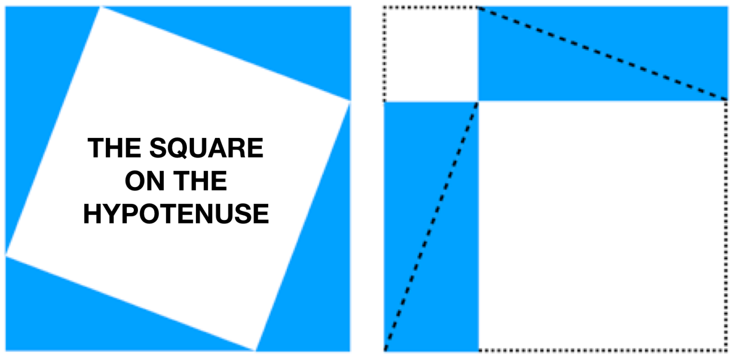
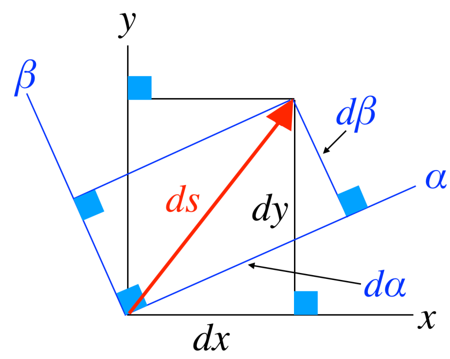
Why it is : that $E = m\,c^2$





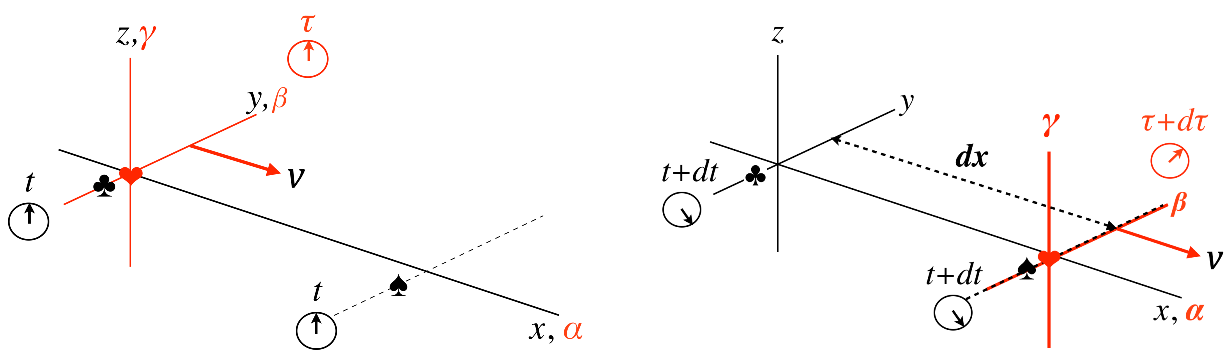
Pythagorean Theorem's *proof* **Invariance** under **Cartesian-Coordinate *rotation***

ALGEBRA invented ⟹ ***rotational***-invariance **EQUATION,** $d\alpha^2 + d\beta^2 = ds^2 = dx^2 + dy^2$

In 1905 and 1908 **the great discovery** by **Albert Einstein** and **Hermann Minkowski**, of

CONSTANT MOTION invariance: $\boldsymbol{(d\alpha^2 + d\beta^2 + d\gamma^2) - d\tau^2 = (dx^2 + dy^2 + dz^2) - dt^2}$

Example: **twice,** ❤ snaps her fingers — as ♣, and then ♠, see ❤ move ***dx*** at constant ***v*** :



❤***-moves*** at ***v*** ⟹ $\boldsymbol{0 + 0 + 0 - d\tau^2 = dx^2 + 0 + 0 - dt^2}$ ⟸ ♠ & ♣ both ***stationary*** in ***x, y, z***

$\boldsymbol{d\tau^2 = dt^2 - dx^2}$ ∴ ❤**'s time *slows!*** But $\boldsymbol{dx = v\,dt}$ and so $d\tau^2 = \boldsymbol{dt^2(1 - v^2)}$ and $\boldsymbol{v}$ $\therefore \leq \boldsymbol{1}$

Gluons and photons move at $\boldsymbol{v = 1}$ light-year/year $\equiv c$ ∴ $\boldsymbol{d\tau} = 0$ ∴ for *them*, **NO** time passes.

[If you *insist* on $c \equiv 3 \times 10^8\ m/s$, then $\boldsymbol{v} \to \boldsymbol{v}/c$] $\left(\frac{dt}{d\tau}\right)^2 = \frac{1}{1-v^2} = 1$ if $v = \mathbf{0}$, but $\to \infty$ if $v \to \mathbf{1}$ $(= c)$

$\left(\frac{d\tau}{d\tau}\right)^2 = \frac{1}{1-v^2} - \frac{v^2}{1-v^2}$ Multiply through by ❤'s *mass*^{2} $m^2 = \left(\frac{m}{\sqrt{1-v^2}}\right)^2 - \left(\frac{m\,v}{\sqrt{1-v^2}}\right)^2 \equiv E^2 - p^2$

$$\boldsymbol{E = \frac{m}{\sqrt{1-v^2}} = m + \frac{1}{2}mv^2 + \frac{3}{8}m\,v^4 + \cdots} \qquad \textbf{— and —} \qquad \boldsymbol{p = \frac{m\,v}{\sqrt{1-v^2}}}$$

BUT: ***if*** ❤ ***doesn't move***, $\boldsymbol{v} = 0$, and ❤'s $\boldsymbol{p} = 0$ **BUT** her $\boldsymbol{E = m}$ $[= mc^2$ ⟵ **iff** $c \neq 1$]

Pythagoras *explains* Newton's $\boldsymbol{p = mv}$, and his $\boldsymbol{E = 1/2\ mv^2}$, and also ***predicts*** $\boldsymbol{E = m\,c^2}$

$\mathit{Mass}\ m = \mathit{Energy}\ E$

1) ***Visual*** proof of the Pythagorean theorem: the two BIG squares are the same size; all ***8*** blue triangles are identical and ∴ have identical areas. So, if you take ***4*** blue triangles away from each of the two BIG squares, the areas that are left must be **exactly** the same in each!

2) ∴ **Algebra** gives a ***correct*** result *!* **and** we see invariance of *ds* under coordinate rotation: the triangle with sides *dx* and *dy* has hypotenuse *ds*. **But so does** the triangle with sides $d\alpha$ and $d\beta$, so ***ds*** is **invariant** under a ***rotation***, e.g. the one that rotated the axes x, y into α, β.

Einstein *discovered* ➞ Invariance under ***translation*** **through-space-&-time.**

3) $d\tau^2 = dt^2 - dx^2$: if $v = \mathbf{0}$, *dx* is **zero**, $d\tau^2 = dt^2$, and $d\tau = dt$. But, if *dx* is **not** zero — if ❤ is *moving* — $\boldsymbol{d\tau}$ *is* ***smaller*** *than* **dt** — it is a ***shorter*** interval of time by ❤'s wristwatch, and so ❤ has aged *less* than ♠ and ♣ have aged*!* Amazing! True? **Yes!** By experiment.

4) Distance = velocity times time: $\boldsymbol{dx} = v\ dt$

5) $d\tau^2 = dt^2\,(1 - v^2)$. The square of *any* number is positive. But if *v* were larger than 1, the right hand side would be negative, and so it **could not** be equal to the left hand side. **So algebra** claims **velocities must be ≤ 1** — just as you can't go north of the North Pole! And indeed, with particle accelerators, we *can* put **huge** energies into single particles, yet we never measure them as going faster than **1** light-year per year — experimental *proof that* ALGEBRA **+** Einstein's Pythagorean idea—his **weird *negative* sign for time**—is **CORRECT**, ***for*** *it predicts* our totally unexpected, unintuitive, result. Wow! We asked, True? **YES, true!**

Notice that velocity has no units! "light-year per year" is just 1 ! However, we traditionally *force* units on velocity, taking the speed of light as being, instead of the natural **1** light-year per year 299,792,458 meters per second. Why do we do that? There's **no reason**, really!

Example: your 60 mph car goes 89 *billionths* of a light-year per year — call it "89 ***blYear***" ?

$$60\,\frac{mile}{hour} \times \frac{hour}{\equiv 3600\ s} \times \frac{\equiv 1.6093440\ km}{mile} \times \frac{s}{\equiv 299{,}792.458\ km} = 0.000{,}000{,}089\,\frac{light\ year}{year}$$

6) If you decide to include c in your equations, simply write $\left(\frac{v}{c}\right)$, in place of *v*, everywhere.

But the simple truth is that c = **1** and on your T-shirt you should really just write ***E* = *m*,** or better yet: ***m* = *E*** . Explain it to your friends, and say that your car goes about **90 *blYear.***

7) In 1905, the **Newtonian** expressions for **momentum** *p* and **Energy** *E* were, forever, *displaced* by Einstein's new **experimentally-correct** expressions for *p* and for *E* :

$$p = \frac{mv}{\sqrt{1-\left(\frac{v}{c}\right)^2}} \qquad \text{and} \qquad E = \frac{m\,c^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \boldsymbol{m}\,c^2 + \frac{1}{2}\boldsymbol{mv^2} + \frac{3}{8}m\left(\frac{v^4}{c^2}\right) + \cdots$$

That last bit ***goes beyond*** **Algebra 1**: the **infinite series** is advanced mathematics that was discovered by **Isaac Newton** and we see that it provides us with the familiar **Newtonian** expression for kinetic energy $\frac{1}{2}\boldsymbol{mv^2}$ so we can, now, notice *how it is* that inside those particle accelerators the kinetic energy can *keep on increasing* ***beyond Newton's*** $\frac{1}{2}\boldsymbol{mv^2}$ **,** *even though* the velocity ***v*** of those particles *never* exceeds *c*. It's all those extra terms ...

8) ^{235}U , hit by a neutron: $n + {}^{235}\mathrm{U} \longrightarrow {}^{236}\mathrm{U} \longrightarrow {}^{92}\mathrm{Kr} + {}^{141}\mathrm{Ba} + [\ \boldsymbol{E} = (m_{\mathrm{U}} - m_{\mathrm{Kr}} - m_{\mathrm{Ba}}) \times \boldsymbol{c^2}\ ] + 3\ n$

Final Exam: "Wait a minute! ❤ sees ♠ and ♣ moving at velocity $-\boldsymbol{v}$ *!* " **Discuss!**

(The exam changes your entire conception regarding the objective reality of the Universe.)