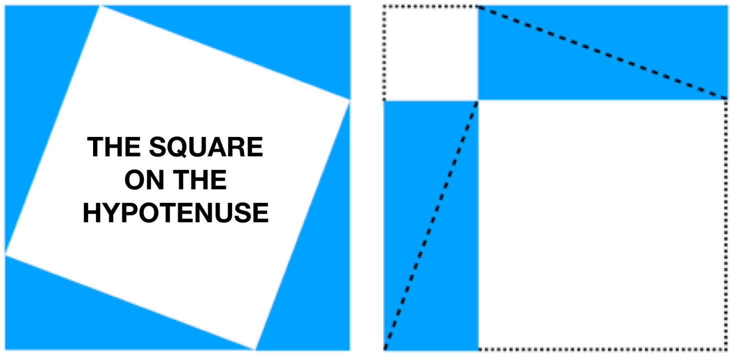
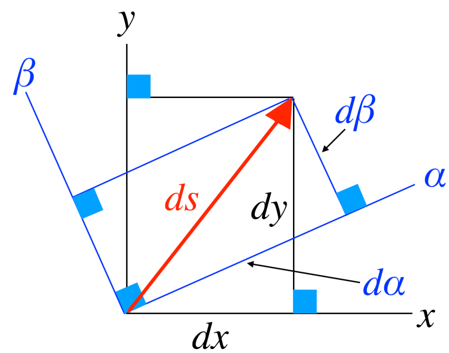
Mass m = *Energy* E





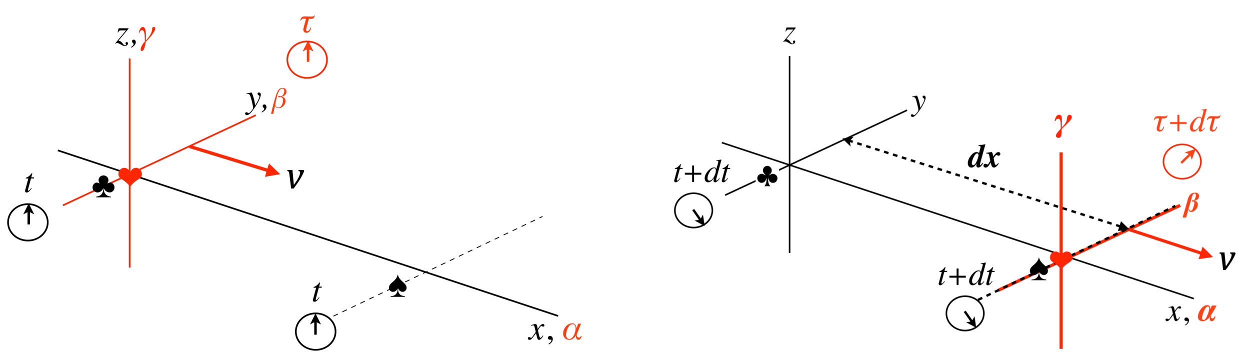
Pythagorean Theorem's *proof*   **Invariance** under **Cartesian-Coordinate *rotation***

ALGEBRA invented ⟹ ***rotational***-invariance **EQUATION:** $d\alpha^2 + d\beta^2 = ds^2 = dx^2 + dy^2$

In 1905 and 1908 **the great discovery** by **Albert Einstein** and **Hermann Minkowski**, of

CONSTANT MOTION invariance: $\boldsymbol{(d\alpha^2 + d\beta^2 + d\gamma^2) - d\tau^2 = (dx^2 + dy^2 + dz^2) - dt^2}$

Example: twice, ❤ snaps her fingers — as ♣, and then ♠, see ❤ move ***dx*** at constant ***v*** :



❤***-moves*** at $\boldsymbol{v}$ ⟹ $\boldsymbol{0 + 0 + 0 - d\tau^2 = dx^2 + 0 + 0 - dt^2}$ ⟸ ♠ & ♣ both ***stationary*** in ***x, y, z***

$\boldsymbol{d\tau^2 = dt^2 - dx^2}$ ∴ **❤'s time *slows!*** But $\boldsymbol{dx = v\, dt}$ and so $d\tau^2 = \boldsymbol{dt^2(1 - v^2)}$ and $\boldsymbol{v}$ ∴$\boldsymbol{\le 1}$

Gluons and photons move at $\boldsymbol{v = 1}$ light-year/year $\equiv c$ ∴ $\boldsymbol{d\tau} = 0$ ∴ for *them*, **NO** time passes.

[If you *insist* on $c \equiv 3 \times 10^8\ m/s$, then $\boldsymbol{v} \to \boldsymbol{v}/c$] $\left(\frac{dt}{d\tau}\right)^2 = \frac{1}{1-v^2} = 1$ if $v = \boldsymbol{0}$, but $\to \infty$ if $v \to \boldsymbol{1}$ $(= c)$

$\left(\frac{d\tau}{d\tau}\right)^2 = \frac{1}{1-v^2} - \frac{v^2}{1-v^2}$ Multiply through by ❤'s *mass*^{2} $m^2 = \left(\frac{m}{\sqrt{1-v^2}}\right)^2 - \left(\frac{m\,v}{\sqrt{1-v^2}}\right)^2 \equiv E^2 - p^2$

$$E = \frac{m}{\sqrt{1 - v^2}} = m + \frac{1}{2}mv^2 + \frac{3}{8}m\,v^4 + \cdots \qquad \text{– and –} \qquad p = \frac{m\,v}{\sqrt{1 - v^2}}$$

BUT: ***if*** ❤ ***doesn't move***, $\boldsymbol{v} = 0$, and ❤'s $\boldsymbol{p} = 0$ **BUT** her $\boldsymbol{E = m}$ $[= mc^2$ ⟵ **iff** $c \neq 1$]

Pythagoras *explains* Newton's *p = mv*, and his ***E* =1/2 *mv*²**, and also ***predicts E = mc²***

ONWARD now to **DYNAMICS**: ***F* = *m a***

❤'s $\boldsymbol{v}$ was **unchanging !** But if, say, **you drop a stone** you see it fall faster, and then, *faster still!* Its downward velocity *increases* with time. And another real-world example: hold a magnet above a small piece of iron: the iron *accelerates upward !* But, unlike the "stone drop" experiment, which works for *any* material, our magnet example *does not:* the magnet will **not** pick up, *e.g.*, paper*!* So whatever causes the rising of the iron is *not* universal, as gravitation is: magnets work with *some stuff*, but not with *all* stuff.

We attribute these observed changes in velocity (that is, these *accelerations*) to magical **FORCES.**

That of course does nothing for us except to give those accelerations a *label*. Nothing more. Still, that is very useful because, over the centuries, **we have only discovered FOUR forces**! So, they *deserve* names! The *first* force was (of course) named the **gravitational force**, investigated by Isaac Newton. Our second force we call the **electromagnetic force**, which combines electricity *with* magnetism. The *remaining* two forces are the **strong force**—it holds atomic nuclei together (against the repulsive electrical force of the protons in the nucleus)—and **the weak force** that reveals its existence by the **observed** spontaneous decay of a neutron into (a proton + an electron + an antineutrino). Both these forces are very short-range, unlike the gravitational and electromagnetic forces.

We have found **equations** that describe the **gravitational** and the **electromagnetic** forces! Recall that

$$\boldsymbol{E} = \frac{\boldsymbol{m}}{\sqrt{\boldsymbol{1 - v^2}}} = \boldsymbol{m} + \frac{\boldsymbol{1}}{\boldsymbol{2}}\boldsymbol{mv^2} + \frac{3}{8} m\, \boldsymbol{v^4} + \cdots \qquad - \textbf{and} - \qquad \boldsymbol{p} = \frac{\boldsymbol{m\,v}}{\sqrt{\boldsymbol{1 - v^2}}} \equiv \gamma\, \boldsymbol{mv}$$

Now, the reason that **we know** that there *are* "forces" in the universe **is because we see velocities *CHANGE***. Those changes are often *dramatic.* Drop a stone! It accelerates rapidly! But we know that even just before it hits the ground, a falling stone is not *really* going very fast at all, despite the acceleration. At, say, 60 *mph*, $\boldsymbol{v}$ is only 0.000000089 $\boldsymbol{c}$, and though that change in $\boldsymbol{v}$ *is* seen by us as a dramatic change in the momentum $\boldsymbol{p}$ of the dropped object, the stone's **total energy** $\boldsymbol{E}$ would hardly change at all: simply because $\boldsymbol{v}^2$ = 0.000000000000007921 which is ≪ **1** ! So we isolate, and name, the **Kinetic Energy, *KE* .**

$$\boldsymbol{KE} \equiv \frac{\boldsymbol{1}}{\boldsymbol{2}}\boldsymbol{mv^2} + \frac{3}{8} m\, \boldsymbol{v^4} + \cdots \cong \frac{\boldsymbol{1}}{\boldsymbol{2}}\boldsymbol{mv^2}$$ **(** the latter, of course, is **Isaac Newton's Energy)**

Force ≡ rate of change of $\boldsymbol{p}$ $\quad \boldsymbol{F} \equiv \frac{dp}{dt} = \frac{d}{dt}(\gamma\, \boldsymbol{mv}) \cong \frac{d}{dt}(\, \boldsymbol{mv}) = m \frac{d}{dt}(\, \boldsymbol{v}) = \boldsymbol{ma}$ **(** again, **Newtonian)**

We now have in our possession | the fundamental equation of **Newtonian Dynamics:** $\boldsymbol{F} = \boldsymbol{ma}$ |

1) Gravitation: $\boldsymbol{F} = \boldsymbol{ma} \cong -\,\boldsymbol{G}\,\frac{\boldsymbol{M\,m}}{\boldsymbol{r^2}}$ (***Newton's gravitation***—*not* Einstein's) gives the force exerted on a body of mass $\boldsymbol{m}$ by the presence at distance $\boldsymbol{r}$ of a body of mass $\boldsymbol{M}$. The minus sign indicates that the force is *attractive*. [We would of course set $\boldsymbol{G}$ = 1 , were it not for the existence of the *other three* forces: they, by experiment, require **different** strengths (by **BIG** factors—gravitation is the *weakest* force, *by far*)]. We note that a planet of mass $\boldsymbol{m}$ gets an acceleration $\boldsymbol{a} = -\,\boldsymbol{G}\,\frac{\boldsymbol{M}}{\boldsymbol{r^2}}$ that is **independent** of its **own** mass.

2) Electric and **Magnetic** forces: $\boldsymbol{F_E} = \boldsymbol{ma} = \boldsymbol{K}\,\frac{\boldsymbol{Q\,q}}{\boldsymbol{r^2}}$ (the force of charges $\boldsymbol{Q}$ and $\boldsymbol{q}$ on each other), and $\boldsymbol{F_M} = \boldsymbol{ma} = \boldsymbol{q\,v} \times \boldsymbol{B}$ is the force on $\boldsymbol{q}$ caused by the magnetic field $\boldsymbol{B}$ created by a current $\mathbf{J}$ of electric charges. If the electric charge density is $\boldsymbol{\rho}$ we find the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ by use of

Maxwell's equations: $$\nabla.\boldsymbol{E} = 4\pi\boldsymbol{\rho} \;\; ; \;\; \nabla.\boldsymbol{B} = 0 \;\; ; \;\; \nabla \times \boldsymbol{E} = -\frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} \;\; ; \;\; \nabla \times \boldsymbol{B} = \frac{1}{c}\left(4\pi\,\mathbf{J} + \frac{\partial \boldsymbol{E}}{\partial t}\right)$$

So now you are **ALL DONE *!*** **(** ***And good news—Maxwell's equations are consistent with Einstein.*)**

YOU are now fully prepared to study physics further: with real understanding!