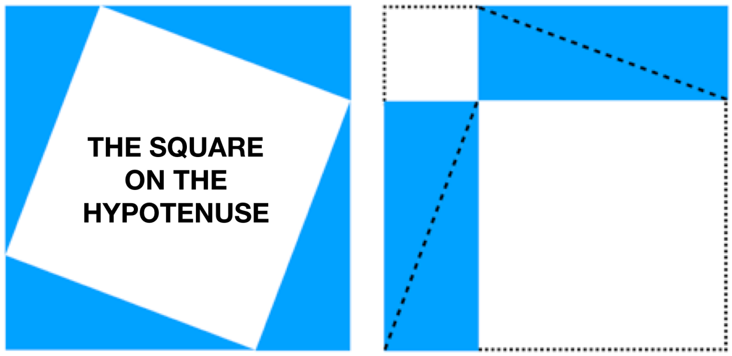
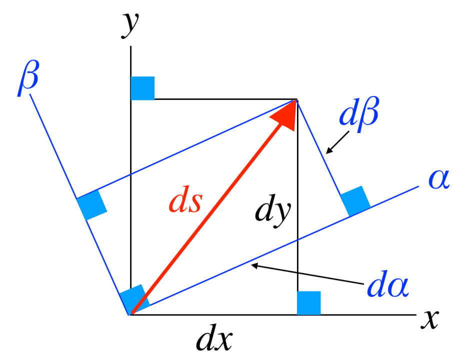
PYTHAGORAS *leads us right to* **Special Relativity,** *and then onward to* **General Relativity**





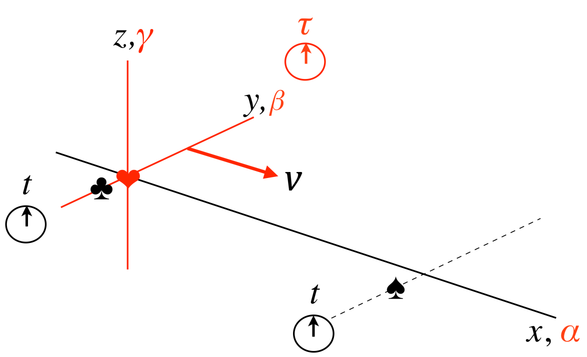
Pythagorean Theorem's proof & its **invariance** under Cartesian-coordinate **rotation.**

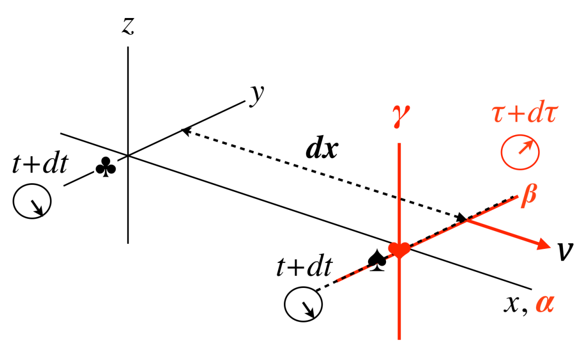
ALGEBRA invented ⟹ **rotational**-invariance-**EQUATION:** $d\alpha^2 + d\beta^2 = ds^2 = dx^2 + dy^2$

Then, *3D,* & then: **ALGEBRA** let us **SUBTRACT** *a* **FOURTH term** *!* **THAT explained TIME :**

CONSTANT– motion invariance: $\boldsymbol{(d\alpha^2 + d\beta^2 + d\gamma^2) - d\tau^2 = (dx^2 + dy^2 + dz^2) - dt^2}$

Example: twice, ❤ snaps her fingers — as ♣, and then ♠, see ❤ move $\boldsymbol{dx}$ at constant $\boldsymbol{v}$:





❤***-moves*** at $\boldsymbol{v}$ ⟹ $\boldsymbol{0 + 0 + 0 - d\tau^2 = dx^2 + 0 + 0 - dt^2}$ ⟸ ♠ & ♣ both ***stationary*** in $\boldsymbol{x, y, z}$

$\boldsymbol{d\tau^2 = dt^2 - dx^2}$ ∴ ❤**'s time *slows!*** But $\boldsymbol{dx = v\, dt}$ and so $\boldsymbol{d\tau^2 = dt^2(1 - v^2)}$ and $\boldsymbol{v} \therefore \leq \mathbf{1}$

Photons (and gluons) move at $\boldsymbol{v} = \mathbf{1}$ light-year/year ∴ $\boldsymbol{d\tau} = 0$ so, for *them*, **NO** time passes.

Cars move at $\boldsymbol{v} = 0.000{,}000{,}089$ (= 60 *mph*) $\left(\frac{dt}{d\tau}\right)^2 = \frac{1}{1-v^2} = 1$ if $v = \mathbf{0}$, but → ∞ as $v \to \mathbf{1}$

$\left(\frac{d\tau}{d\tau}\right)^2 = \frac{1}{1-v^2} - \frac{v^2}{1-v^2}$ Multiply through by ❤'s *mass*² $m^2 = \left(\frac{m}{\sqrt{1-v^2}}\right)^2 - \left(\frac{m\,v}{\sqrt{1-v^2}}\right)^2 \equiv E^2 - p^2$

$$\boldsymbol{E = \frac{m}{\sqrt{1-v^2}} \quad \left(= m + \frac{1}{2}mv^2 + \cdots\right)}$$

— and —

$$\boldsymbol{p = \frac{mv}{\sqrt{1-v^2}} \quad (= mv + \cdots)}$$

if ❤ ***doesn't move***, $\boldsymbol{v} = \mathbf{0}$, and ❤'s $\boldsymbol{p} = \mathbf{0}$ **BUT** ❤ still retains **Energy** $\boldsymbol{E}$ = ***mass*** $\boldsymbol{m}$

We invented **algebra** as a convenience—but: algebra *seems to* ***understand*** the Universe!

Special Relativity: $ds^2 = dx^2 + dy^2 + dz^2 - dt^2$

Next up: **GENERAL Relativity,** yielding Cosmology, and also Black Holes

SR said that: $ds^2 = dx^2 + dy^2 + dz^2 - dt^2$ Optionally ➔ **Spherical-polar**: $ds^2 = dr^2 + r^2 d\theta^2 + r^2 sin^2\theta\, d\varphi^2 - dt^2$

GR needs *more than* **numbers** *!* **GR** needs new mathematical **entities** that are SQUARE ARRAYS of numbers:

$$g^\mu_\nu = \begin{pmatrix} g^1_1 & g^2_1 & g^3_1 & g^4_1 \\ g^1_2 & g^2_2 & g^3_2 & g^4_2 \\ g^1_3 & g^2_3 & g^3_3 & \mathbf{g^4_3} \\ g^1_4 & g^2_4 & \mathbf{g^3_4} & g^4_4 \end{pmatrix} = e.g., \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \mathbf{0} \\ 0 & 0 & \mathbf{0} & \mathbf{1} \end{pmatrix}$$

General Relativity is $4 \times 4 = 16$ coupled equations. **GR** gives us **Cosmology: 1** is Newton's gravitation; the *three* 1's are Pressure. (AND, **GR** gives us **Black Holes**)

Einstein's **idea** was that Newton's Law of Universal Gravitation **might** be a result of spacetime being **curved** (*somehow !*) by the presence of masses *e.g.* the **Sun**—resulting in Earth ***circling*** the Sun. The great man created **GR** in 1916, in wartime Germany. Here it is :

$$\textit{spacetime-curvature} \rightarrow \quad R^\mu_\nu - \frac{1}{2}\, g^\mu_\nu\, R \left(+\, g^\mu_\nu\, \boldsymbol{\Lambda}\right) = \frac{8\pi G}{c^4}\, T^\mu_\nu \quad \leftarrow \textit{\textbf{Mass}}\ \&\ \textit{\textbf{Energy}}$$

Mass & Energy is **conserved:** its **derivative is zero**. Observations today say we must *add in* a **constant :** $\boldsymbol{\Lambda}$

spacetime-curvature $R^\mu_\nu - \frac{1}{2}\, g^\mu_\nu\, R$ is ***also* conserved** (in the same sense) ← *took* Einstein ***work*** to find it *!*

$\boldsymbol{\Lambda}$ *NOW* dominates **cosmology**: a length ℓ of *space-itself* is **exploding** (*even as you read this*): $\ell(t) \propto e^{\sqrt{\frac{\Lambda}{3}}\, t}$

Einstein himself was **never** able to find **even *one-single-solution*** for those famous Field Equations ! He **DID** *calculate* ***manually*** what they predicted for the precession of the planet Mercury: **and it *agreed* with *observation !*** **and:** it did **not** agree with the **Newtonian prediction*!*** Einstein had heart palpitations for a week! He *published!* His greatest triumph!

Einstein's 1915 paper was in a German physics journal—which arrived at a German soldier, **Karl Schwarzschild**, on the Eastern Front. (I knew Karl's son!) And, Karl **found** a solution!

THIS is the famous **Schwarzschild Solution** to $R^\mu_\nu = 0$ (the **vacuum** equations)

$$ds^2 = +\, \mathbf{g^1_1}\, \frac{dr^2}{\left(1 - \frac{2\boldsymbol{m}}{\boldsymbol{r}}\right)} + \mathbf{g^2_2}\; r^2 d\theta^2 + \mathbf{g^3_3}\; r^2 sin^2\theta\; d\varphi^2 - \mathbf{g^4_4}\left(1 - \frac{2\boldsymbol{m}}{\boldsymbol{r}}\right) dt^2$$

— ***WARPED PYTHAGORAS!*** —

Karl's solution is for the case of a SINGLE POINT-MASS causing the curvature. In 1917, the idea of **huge point masses** was ridiculous—but white dwarf stars were found, and then neutron stars were found ... and then, at last, **black holes were found.** The galaxy M87 has, at its center, a rotating thin **ring** of **6 billion** solar masses! Just empty spacetime elsewhere.

(... how do we include **rotation**? The **Kerr black hole** has terms including $g^3_4 = g^4_3 = 1$)

r* is *your distance from the **point mass.** For a 20-solar-mass black hole, ***m*** = ~**20** miles. Now, notice **what happens** as you ***fall in:*** when the moment comes that ***r*** falls to **40** miles, the first term (+) changes over to TIME (**-**) but it is, nevertheless still — your *fall-direction!* **Why** can't you get out of the black hole ? **It 's only because you *cannot* go back in TIME *!***

The Gaussian Curvature of spacetime near, at, and inside, a black hole is: ZERO *!*

The possibility of **Black Holes** in our universe was **predicted** by **Pythagoras:** as advanced by Albert Einstein and by Hermann Minkowski, and, finally, by Karl Schwarzschild *!*