POSTULATES OF QUANTUM MECHANICS

From "Dicke and Wiitke"

POSTULATE 1. It is assumed for a system consisting of a particle moving in a conservative field of force (produced by an external potential), that there is an associated wave function, that this wave function determines everything that can be known about the system, and that it is a single-valued function of the coordinates of the particle and of the time.* In general it is a complex function, and may be multiplied by an arbitrary complex number without changing its physical significance.

POSTULATE 2. With every physical observable (the energy of the system, the x-position coordinate of the particle, etc.) there is associated an operator. Denote by Q the operator associated with the observable q. then a measurement of q gives a result which is one of the eigenvalues of the eigenvalue equation:

$$Q\psi_n = q_n\psi_n \qquad (1)$$

This measurement constitutes an interaction between the system and the measuring apparatus. If the state function was ψ_n prior to the measurement, the result q_n is certain to be obtained from an exact measurement of the observable associated with the operator Q. If initially the wave function is not an eigensolution of Eq. (1), it is impossible to predict with certainty which of several possible results will be obtained. However, if the result q_n is obtained, the interaction changes the state of the system to the state described by the function ψ_n. This is equivalent to the condition that a measurement be repeatable: a measurement giving a result q_n will, if repeated immediately, give with certainty the same result.

POSTULATE 3. Any operator associated with a physically measurable quantity is Hermitian.

POSTULATE 4. The set of functions ψ_j which are eigenfunctions of the eigenvalue equation

$$Q\psi_j = q_j\psi_j \qquad (2)$$

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* Since $|\psi|^2$, and not ψ itself, has been seen to be the quantity of measurable physical significance, the necessity for the assumption of *single-valuedness* is not *a priori* obvious. However, various mathematical difficulties arise if the single-valuedness postulate is abandoned, and so it will be retained for the purposes of this book. For a more detailed discussion of this point see: W. Pauli, *Die allgemeinen Prinzipien der Wellenmechanik*, J. W. Edwards, Ann Arbor, Mich., 1947, p. 126 (reprinted from Handbuch der Physik, 2nd ed., vol 24, part 1); J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics*, John Wiley and Sons, New York, 1952, Appendix A, footnotes on p. 783 and p. 787.

form, in general, an infinite set of linearly independent functions. A linear combination of these functions of the form

$$\psi = \sum_j c_j \psi_j \qquad (3)$$

can be used to express an infinite number of possible functions. It might be expected that this infinite set of linearly independent functions could be used to expand any arbitrary function ψ; actually this assumption is more stringent than is necessary. It will be assumed only that the infinite set of functions formed by the eigenfunctions of any operator playing a role in quantum mechanics can be used to expand a wave function which is a suitable physical wave function. Questions involved in the possibility of expansion of a particularly badly behaved function will not be considered. It is specifically assumed that if ψ is a physically acceptable wave function, it can be expanded in eigenfunctions of any observable of the system.

POSTULATE 5. If a system is described by a wave function ψ, the expectation value of any observable q with corresponding operator Q is given by

$$\langle q \rangle = \int \overline{\psi} Q \psi \, dr \qquad (4)$$

POSTULATE 6. The development in time of the wave function ψ, given its form at an initial time and assuming the system is left undisturbed, is determined by the Schrödinger equation

$$H\psi = i\hbar \frac{\partial}{\partial t}\psi \qquad (5)$$

where the Hamiltonian operator H is formed from the corresponding classical Hamiltonian function by substituting for the classical observables their corresponding operators.

POSTULATE 7. The operators of quantum theory are such that their commutators are proportional to the corresponding classical Poisson brackets according to the prescription

$$[Q,R] \equiv (QR - RQ) \Leftrightarrow i\hbar\{q,r\} \qquad (6)$$

Where $\{q,r\}$ is the classical Poisson bracket for the observables q and r. The variables, if any, in the Poisson bracket are to be replaced by operators.

Two observations should be made in connection with this postulate. The coordinates and momenta must be expressed in Cartesian coordinates. Also, in certain cases, ambiguities can arise in the order of noncommuting factors. These can often be resolved by remembering that the operator must be Hermitian. Because of these limitation and ambiguities, this “postulate” must be regarded more as a helpful guide than as a basic postulate of quantum mechanics.