# Does the Inertia of a Body Depend on its Energy Content ?

**by A. Einstein**

The results of an electrodynamic study [1]) recently published by me in these annals lead to a very interesting conclusion, which is to be derived here.

I used the Maxwell-Hertz equations for empty space as a basis along with the Maxwell expression for the electromagnetic energy of space and also the principle:

The laws, by which the states of physical systems change, are independent of which of two coordinate systems that are in a uniform parallel translation movement relative to one another these state changes are referred to ( Principle of Relativity ).

Based on these Principles [2]), I derived the following result, among other things:

A system of plane waves of light, referred to the coordinate system $(x, y, z)$, has the energy $\ell$ ; the beam direction (wave normal) forms the angle $\varphi$ with the $x$-axis of the system. If one introduces a new coordinate system $(\xi, \eta, \zeta)$, which is understood to be in uniform parallel translation with respect to the system $(x, y, z)$ and whose origin moves along the $x$-axis with speed $v$, then the mentioned amount of light – measured in the system $(\xi, \eta, \zeta)$ – has the energy:

$$\ell^* = \ell \, \frac{1 - \frac{v}{V} \cos\varphi}{\sqrt{1 - \left(\frac{v}{V}\right)^2}} \,,$$

where $V$ is the speed of light. We will make use of this result in the following.

Now consider a body at rest in the system $(x, y, z)$, the energy of which – in relation to the system $(x, y, z)$ – is $E$. Relative to the system $(\xi, \eta, \zeta)$ moving with velocity $v$ as above, let the energy of the body be $H$.

This body sends light waves of the energy $L / 2$ (measured relative to $(x, y, z)$ ) and at the same time an equally large amount of light in the opposite direction, in a direction forming the angle $\varphi$ with the $x$-axis. Here the body remains at rest in relation to the system $(x, y, z)$. The energy principle must apply to this process, namely (according to the principle of relativity) with regard to both coordinate systems. If we call $E$ or $H$ the energy of the body when the light emission is measured relative to the system $(x, y, z)$ or $(\xi, \eta, \zeta)$, then using the above relation we get:

$$E_0 = E_1 + \left[ \frac{L}{2} + \frac{L}{2} \right] ,$$

$$H_0 = H_1 + \left[ \frac{L}{2} \, \frac{1 - \frac{v}{V} \cos\varphi}{\sqrt{1 - \left(\frac{v}{V}\right)^2}} + \frac{L}{2} \, \frac{1 + \frac{v}{V} \cos\varphi}{\sqrt{1 - \left(\frac{v}{V}\right)^2}} \right] = H_1 + \frac{L}{\sqrt{1 - \left(\frac{v}{V}\right)^2}}$$

Subtracting these equations gives:

$$(H_0 - E_0) - (H_1 - E_1) = L \left\{ \frac{1}{\sqrt{1 - \left(\frac{v}{V}\right)^2}} - 1 \right\}$$

1) A. Einstein, Ann. d. Phys. 17. p. 891. 1905.
2) The Principle of constancy of the speed of light is naturally contained in Maxwell's equations.

The two differences in the form $H - E$ appearing in this expression have simple physical meanings. $H$ and $E$ are energy values of the same body, related to two coordinate systems moving relative to one another, with the body resting in one system (system $(x, y, z)$). It is therefore clear that the difference $H - E$ can differ from the kinetic energy $K$ of the body in relation to the other system (system $(\xi, \eta, \zeta)$) only by an additive constant $C$, which depends on the choice of the arbitrary additive constants of the energies $H$ and $E$. So we can put:

$$H_0 - E_0 = K_0 + C$$
$$H_1 - E_1 = K_1 + C$$

because $C$ does not change during the emission of light. So we get:

$$K_0 - K_1 = L\left\{ \frac{1}{\sqrt{1 - \left(\frac{v}{V}\right)^2}} - 1 \right\}$$

The kinetic energy of the body in relation to $(\xi, \eta, \zeta)$ decreases as a result of the emission of light, namely by an amount that is independent of the qualities of the body. The difference $K_0 - K_1$ also depends on the speed as does the kinetic energy of the electron.

Neglecting fourth and higher order magnitudes we can put:

$$K_0 - K_1 = \frac{L}{V^2}\frac{v^2}{2}$$

From this equation it follows immediately:

If a body emits the energy $L$ in the form of radiation, its mass is reduced by $L / V^2$. Here it is evidently inevitable that the energy withdrawn from the body goes into the energy of the radiation, so that we are led to the general conclusion:

The mass of a body is a measure of its energy content; if the energy changes by $L$, the mass changes in the same sense by $L/9 \times 10^{20}$ if the energy is measured in ergs and the mass in grams.

It cannot be ruled out that in bodies whose energy content is highly variable (*e.g.* in the case of radium salts), testing of the theory will succeed.

If the theory is true, radiation imparts inertia between the emitting and absorbing bodies.

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$$\Delta M = \frac{L}{c^2}$$

40 years later: Hiroshima and Nagasaki.