Why it is: that $E = m \ c^2$

**Pythagorean Theorem’s proof**  Invariance under Cartesian-Coordinate rotation

ALGEBRA invented $\Rightarrow$ rotational-invariance EQUATION, $da^2 + dB^2 = ds^2 = dx^2 + dy^2$

In 1905 and 1908 the great discovery by Albert Einstein and Hermann Minkowski, of

**CONSTANT MOTION** invariance: $(da^2 + dB^2 + dy^2) - d\tau^2 = (dx^2 + dy^2 + dz^2) - dt^2$

Example: twice, ♥ snaps her fingers — as ♠, and then ♠, see ♥ move $dx$ at constant $v$:

$\Delta\tau^2 = \Delta t^2 - \Delta x^2 : \spadesuit$'s time slows! But $dx = v \ dt$ and so $\Delta\tau^2 = \Delta t^2 (1 - v^2)$ and $v : \leq 1$

Gluons and photons move at $v = 1$ light-year/year $\equiv c : \Delta\tau = 0 :$ for them, NO time passes.

[If you insist on $c \equiv 3 \times 10^8 \ m/s$, then $v \rightarrow v/c$] $\left(\frac{dt}{d\tau}\right)^2 = \frac{1}{1-v^2} = 1$ if $v = 0$, but $\rightarrow \infty$ if $v \rightarrow 1 (= c)$

$\left(\frac{d\tau}{dt}\right)^2 = \frac{1}{1-v^2} - \frac{v^2}{1-v^2}$ Multiply through by ♥'s mass $^2 \ m^2 = \left(\frac{m}{\sqrt{1-v^2}}\right)^2 - \left(\frac{mv}{\sqrt{1-v^2}}\right)^2 \equiv E^2 - p^2$

$$E = \frac{m}{\sqrt{1-v^2}} = m + \frac{1}{2}mv^2 + \frac{3}{8}m \ v^4 + \cdots \quad \text{and} \quad p = \frac{mv}{\sqrt{1-v^2}}$$

**BUT:** If ♥ doesn't move, $v = 0$, and ♥'s $p = 0$ BUT her $E = m \ [= mc^2 \ \leftarrow \text{iff } c \neq 1 \]$  

Pythagoras explains Newton's $p = mv$, and his $E = 1/2 m \ v^2$, and also predicts $E = m \ c^2$
Mass \( m = \text{Energy } E \)

1) **Visual proof** of the Pythagorean theorem: the two BIG squares are the same size; all 8 blue triangles are identical and \( \vdash \) have identical areas. So, if you take 4 blue triangles away from each of the two BIG squares, the areas that are left must be **exactly** the same in each!

2) \( \vdash \) **Algebra gives a correct result!** and we see invariance of \( ds \) under coordinate rotation: the triangle with sides \( dx \) and \( dy \) has hypotenuse \( ds \). But so does the triangle with sides \( da \) and \( db \), so \( ds \) is invariant under a **rotation**, e.g. the one that rotated the axes \( x, y \) into \( \alpha, \beta \).

**Einstein discovered \( \rightarrow \) Invariance** under **translation through-space-\&-time**.

3) \( dt^2 \neq dx^2 - d\gamma^2 \): if \( v = 0 \), \( dx \) is **zero**, \( dt^2 = dx^2 \), and \( dt = dt \). But, if \( dx \) is **not** zero — if \( \heartsuit \) is moving — \( dt \) is smaller than \( dt \) — it is a **shorter** interval of time by \( \heartsuit \)'s wristwatch, and so \( \heartsuit \) has aged less than \( \spadesuit \) and \( \spadesuit \) have aged! Amazing! True? Yes! By experiment.

4) Distance = velocity times time: \( dx = v \, dt \)

5) \( dt^2 = dt^2 \) (1 - \( v^2 \)). The square of **any** number is positive. But if \( v \) were larger than 1, the right hand side would be negative, and so it **could not** be equal to the left hand side. **So algebra** claims velocities must be \( \leq 1 \) — just as you can’t go north of the North Pole! And indeed, with particle accelerators, we can put huge energies into single particles, yet we never measure them as going faster than 1 light-year per year — experimental proof that ALGEBRA + Einstein’s Pythagorean idea—he’s **weird negative sign for time**—is CORRECT, for it predicts our totally unexpected, unintuitive, result. Wow! We asked, True? Yes, true!

Notice that velocity has no units! “light-year per year” is just 1! However, we traditionally **force** units on velocity, taking the speed of light as being, instead of the natural 1 light-year per year \( 299,792,458 \) meters per second. Why do we do that? **There’s no reason, really!**

**Example:** your 60 mph car goes 89 **billionths** of a light-year per year — call it “89 blYear”?

\[
60 \, \text{mile/hour} \times \frac{\text{hour}}{3600 \, s} \equiv 1.6093440 \, \text{km/mile} \times \frac{s}{299,792,458 \, \text{km/year}} = 0.000,000,089 \frac{\text{light-year}}{\text{year}}
\]

6) If you decide to include \( c \) in your equations, simply write \( \left( \frac{v}{c} \right) \), in place of \( v \), everywhere.

But the simple truth is that \( c = 1 \) and on your T-shirt you should really just write \( E = m \), or better yet: \( m = E \). Explain it to your friends, and say that your car goes about 90 blYear.

7) In 1905, the **Newtonian** expressions for momentum \( p \) and Energy \( E \) were, forever, displaced by Einstein’s new **experimentally-correct** expressions for \( p \) and for \( E \):

\[
p = \frac{mv}{\sqrt{1 - \left( \frac{v}{c} \right)^2}} \quad \text{and} \quad E = \frac{mc^2}{\sqrt{1 - \left( \frac{v}{c} \right)^2}} = mc^2 + \frac{1}{2} mv^2 + \frac{3}{8} m \left( \frac{v^4}{c^2} \right) + \ldots
\]

That last bit **goes beyond Algebra 1**: the **infinite series** is advanced mathematics that was discovered by **Isaac Newton** and we see that it provides us with the familiar Newtonian expression for kinetic energy \( \frac{1}{2} \, mv^2 \) so we can, now, notice how it is that inside those particle accelerators the kinetic energy can **keep on increasing beyond Newton’s \( \frac{1}{2} \, mv^2 \)**, even though the velocity \( v \) of those particles never exceeds \( c \). It’s all those extra terms …

8) \( ^{235}\text{U} \), hit by a neutron: \( n + ^{235}\text{U} \rightarrow ^{236}\text{U} \rightarrow ^{92}\text{Kr} + ^{141}\text{Ba} + [ \, E = (m_v - m_c - m_n) \times c^2 ] + 3 \, n \)

**Final Exam:** “Wait a minute! \( \heartsuit \) sees \( \spadesuit \) and \( \spadesuit \) moving at velocity \( -v \)!” Discuss!

(The exam changes your entire conception regarding the objective reality of the Universe.)